Homework assignment-CORRECTION

Due Monday, Feb 13th 2019.

Correction: Equation (1) has been corrected. Question 1 (a), (b) and (c) are adjusted. If you already finished problem 1, you can submit your answer as it is and will be graded based on previous equation (1).

Problem 1: Thermal radiation

We briefly learned about black-body radiation in class. A simple way to understand this is that any object with non-zero temperature (T, unit: Kelvin) will radiate EM-wave (thermal radiation). The black body spectral radiance (power per emitting area per solid angle per unit wavelength) is given by

$$u(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}, \tag{1}$$

where $h = 6.62 \times 10^{-34}\ m^2kg/s$ is Plank constant, $k = 1.38 \times 10^{-23}\ m^2kg/s^2/K$ is Boltzmann constant and c is speed of light.

(a) At a certain temperature T, there exists a frequency λ_m that has the strongest emission intensity according to equation (1). Show that ν_m satisfies this equation: $\frac{hc}{\lambda kT} = 5(1 - e^{-hc/\lambda kT})$. (Hint: same as finding the maximum of a function in calculus.)

(b) One solution of this equation is $hc/\lambda kT = 0$. Can you use Mathematica/Matlab to find the other numerical value of $hc/\lambda kT$ that satisfies the equation above in (a)?

(c) We can write the solution in (b) as $\lambda_m = b/T$. What is the numerical value of constant b? This equation is also called Wien's displacement law. It was discovered experimentally before Plank's black body radiation equation.

(d) The radiation spectrum of our sun has maximum emission at wavelength 500 nm. What is the corresponding temperature of the sun? (This corresponds to the surface temperature of the sun)

(e) Google sun's surface temperature. Is it close to your result? Fun fact: This is how people determine the temperature of star. And from the temperature, physicists can derive the star's mass and their radius.

(f) What frequency has the strongest emission at room temperature (300K)?

(g) Prof. Robert Weikle's group is famous for EM-wave experiments in THz range. If they want to detect their 10 THz signal (EM-wave with frequency 10 THz) at single photon level, can they do it in room temperature? (A successful single photon experiment means: if they produce one photon from their setup, their detector will receive one photon with a reasonable amount of noise.)

(h) Bonus (interesting facts): Black hole is an interesting object in physics. At the ending phase of some stars, they can evolve into black holes. The density of black hole is so high that it creates huge gravitational force that sucks everything around it into the black hole. Even light cannot escape so people once believed the black hole will look absolutely black (that's why people call it black hole). However, black hole has non-zero temperature. When you compress so many materials into a small area it must be pretty hot (just think about the bike pump when you pump air into the tire). So whether black hole violates black-body radiation (and thus second law of thermodynamics) became a well-known question in 1970s.

Prof. Stephen Hawking was the first physicist to show that, the uncertainty principle in quantum mechanics will enable black hole to output thermal radiation, and the radiation will have the exact spectrum as equation (1). This is the famous Hawking radiation. If you are interested in what the black hole "looks like", check the movie "Interstellar". There's an image of a black hole in the movie, and it was generated by numerical calculation of general relativity and quantum mechanics using supercomputer in Prof. Kip Thorne's group. Prof. Kip Thorne is Prof. Stephen Hawking's close friend, and he won the Nobel Prize in 2017 for the detection of gravitational wave.

Problem 2: Quantum well

Quantum well can be considered as man-made atoms/molecules. It has a series of energy levels. Unlike nature atoms/molecules, the energy levels of quantum well can be controlled by engineering the physical dimensions of the quantum well. Quantum well is now widely used in lasers and optical amplifiers.

The figure below shows a simplified configuration of one-dimensional quantum well. An electron is trapped in a potential well with infinite potential barriers.

(a) If electron is wave, what boundary condition does the wave satisfy in the quantum well?

(b) Using the boundary condition, what are the possible wavelengths of the wave? You can assume the wave has the form of sine or cosine wave (not necessary, but it will simplify the analysis).

(c) What is the corresponding momentum (p) for these wavelengths?

(d) The energy of the electron can be expressed as $E = p^2/2m$. What are the energy levels for the electron?

(e) Notice energy of the ground state (lowest energy state) is not zero. Is this consistent with uncertainty principle? (This is a bonus question as we haven't officially introduce uncertainty principle yet.)

(f) Can you predict the emission spectrum of this quantum well? (Just the wavelength or frequency, no need to work on intensity. Bonus: You can derive the relative intensity of these spectrum lines by assuming the electron is in thermal equilibrium. This only requires using some simple equations in the

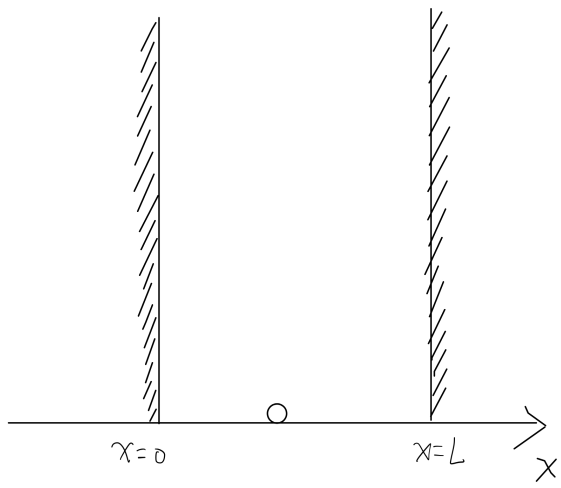

Figure 1: One dimensional quantum well.

note.)

(g) Usually, people excite the electron in the quantum well into its second lowest energy state (first excited state), and the electron will decay to the lowest energy state (ground state), and emit a photon. Suppose we want the color of the photon to be green (photon wavelength equals to 500 nm), what should be the length of quantum well (L)? You will need to look up the mass of the electron.

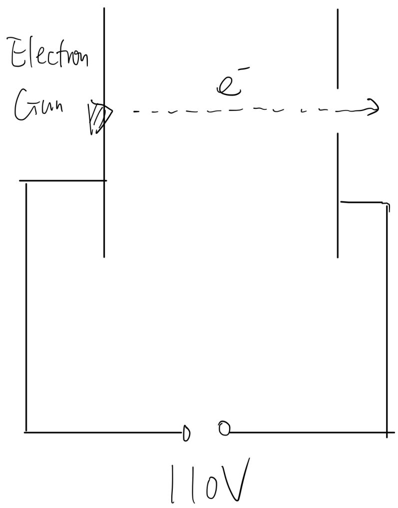

Figure 2: Ted's electron acceleration setup.

Problem 3: De Broglie wavelength

I want everyone to calculate your own De Broglie wavelength, but I can't ask you to submit the answer, as this will clearly offend your privacy. So let's do it to our imaginary friend, Ted. Ted weights 50 kg, and he is good at sprinting. He can run up to 10 m/s (close to Olympic champinion level).

(a) What is the De Broglie wavelength of Ted when he sprints?

(b) Suppose Ted wants to use himself to do double slit interference experiment. The double slit experiment would require the slit to be narrower than the wavelength. Do you think it's possible to build such a narrow slit?

(c) Now Ted took your advice and use electron, instead of himself for the double slit interference experiment. He already has a setup for Young's double slit experiment for light at 500 nm. Now he adds a setup to accelerate the electron by connecting the 110 Volt standard electric outlet to a pair of parallel metal sheets. What is the momentum of the electron after acceleration? What is the De Broglie wavelength?

(d) If Ted wants to create electron interference pattern that is the same as the pattern of light at 500 nm, what voltage he should apply to accelerate his electrons?

(e) Do you think he can do this experiment at room temperature? Hint: At room temperature, on average each electron will have kinetic energy of $1.5kT$, where $k = 1.38 \times 10^{-23}$ $m^2kg/s^2/K$ is Boltzmann constant, T is temperature in Kelvin. This kinetic energy will cause random motion of the electron, and can be considered as a noise source for the experiment.